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ADHESIVE WEAR: GENERALIZED RABINOWICZ' CRITERIA

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Abstract. *In a recent paper in Nature Communications, Aghababaei, Warner and Molinari [5] used quasi-molecular simulations to confirm the criterion for formation of debris, proposed in 1958 by Rabinowicz [4]. The work of Aghababaei, Warner and Molinari improves our understanding of adhesive wear but at the same time puts many new questions. The present paper is devoted to the discussion of possible generalizations of the Rabinowicz-Molinari criterion and its application to a variety of systems differing by the interactions in the interface and by the material properties (elastic and elastoplastic) and structure (homogeneous and layered systems). A generalization of the Rabinowicz-Molinari criterion for systems with arbitrary complex contact configuration is suggested which does not use the notion of "asperity".*

Key words: *Plasticity, Adhesion, Critical Length, Adhesive Wear, Layered Systems, Functionally Gradient Materials, Rabinowicz' Criterion*

1. INTRODUCTION

Among the basic tribological phenomena of contact, adhesion, friction, lubrication and wear, wear remains the least scientifically understood. This may be due to the complexity and diversity of the processes leading to wear. In particular, wear is not a purely contact mechanical phenomenon, but necessarily also includes fracture phenomena within the material and material transport with the resulting very broad problem of the "third body". At the same time, wear remains one of the most important tribological phenomena in practice, affecting all aspects of our lives and current technologies. Wear significantly determines the life time of mechanical systems; it is a key factor in matters of technical safety. Wear not only affects machines and mechanical constructions, it is e.g. also an unsatisfactorily solved problem in medicine: Many implants, especially artificial joints, have to be replaced after approx. 10 years. Wear is also an important issue in terms of

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emission of wear particles into the environment (such as brakes and tires). In all of these areas, very great efforts are undertaken to get wear under control. Up to now, this has largely been done in a purely empirical way.

The most common basis for wear calculations is formed by a law which was formulated in 1953 by Archard [1] and bears his name. According to Archard's law, the wear volume is proportional to the sliding length, the normal force, and inversely proportional to the hardness of the material. The coefficient of proportionality is called wear coefficient. But the devil is in precisely this "coefficient", because empirically measured values of the adhesive coefficient of wear can differ by 7 decimal orders of magnitude [2], which invalidates the influence of hardness. Accordingly, general recommendations made on the basis of the Archard's law have a very limited scope of applicability. For example, there is a widely spread opinion that the higher the hardness, the lower the wear, since the hardness stands in the denominator of the Archard's equation. However, Kragelsky [3] formulated an almost exactly opposite principle for minimizing wear – the principle of a positive hardness gradient, which states that the surface layers must be softer than the lower layers, otherwise catastrophic wear occurs. These two statements, which seemingly exclude each other, both have empirical confirmation, which only emphasizes that the physics of the wear process has so far been poorly understood at its core.

In the last few decades, however, some ideas have been collected which shed new light on the physics of wear. Thus, Molinari with collaborators picked up and confirmed an old idea by Rabinowicz (1958), [4], about the physical mechanism that determines the size of the wear particles and also controls the transition from mild to catastrophic wear. In the criterion of Rabinowicz and the theory of Molinari et al. based on that [5], it is the interplay of plasticity and adhesion, which leads to the appearance of a characteristic length: If a micro contact is smaller than the characteristic length, it is plastically deformed; if it is larger than the characteristic length, wear particles form. The existence of these two scenarios has been independently confirmed by Popov and Dimaki using the method of movable cellular automata and has been observed in molecular dynamics simulations [6]. Combined with advanced numerical simulation methods of contact between rough surfaces [7], this new understanding advances the old idea of Rabinowicz to a new paradigm [8].

The Rabinowicz-Molinari criterion applies to homogeneous systems. However, the surface region of contacts in most low-wear technical systems is not homogeneous. Through the work of Gerve et al. [9] and in the last decade, especially by M. Scherge and co-workers [10], the role of very thin chemically modified surface layers has been demonstrated for systems with "minimal wear" (including, for example, combustion engines). Further, the Rabinowicz-Molinari criterion uses the notion of "asperity". However, the development of the contact mechanics of rough surfaces has shown that this notion is poorly defined. Thus, it is important to search for formulations of the same physical principles without using the notion of asperity. In the present paper, the basic physical idea underlying the criterion of Rabinowicz-Molinari will be applied to heterogeneous media. Further, "asperity-free" concepts will be discussed.

2. RABINOWICZ' CRITERION FOR FORMATION OF WEAR DEBRIS

2.1. Original Rabinowicz' criterion for homogeneous media

We start with the reproduction of the well-known derivation of the Rabinowicz criterion [2, 4, 11] for a homogeneous medium. If two micro heterogeneities collide and form a welded bridge, as suggested by Bowden and Tabor [12], (see Fig. 1) they are plastically deformed, and the maximum stress that can be achieved is of the order of the hardness of the material. In this state, the stored elastic energy is proportional to the third power of contact size:

$$U_{el} \approx \frac{\sigma_0^2}{2G} D^3 . \tag{1}$$

This energy can relax by creating a wear particle. The process of detaching a wear particle can, however, only occur if the stored elastic energy exceeds the energy

$$U_{adh} \approx \Delta w \cdot D^2 \tag{2}$$

which is needed to create new free surfaces (where Δw is the work of adhesion per unit area). It follows that only particles larger than some critical size can be detached:

$$D > \frac{2G \cdot \Delta w}{\sigma_0^2} . \tag{3}$$

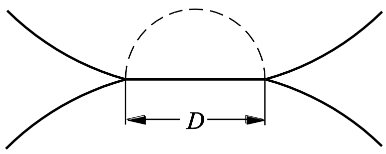

Fig. 1 Welded joint of size *D* created due to contact and shear of two asperities.

Note that the Eq. (3) predicts only the existence of a lower bound of the size of wear particles. Thus, there also should be some mechanism suppressing the appearance of too large particles. Rabinowicz did not make any suggestions for such a mechanism. However, a possible mechanism could be very simple and follow from the same Eq. (3). Indeed, as noticed in [13], if some particle has size much larger than (3), it is energetically favorable for it to disintegrate into two smaller ones. This process can only continue until the critical size (3) is reached. Thus, the wear particles should all have a size of the same order of magnitude as the critical length given by Eq. (3).

2.2. Modified Rabinowicz' criterion for frictional interaction in the interface

Consider the same asperity contact as shown in Fig. 1, but assume now that the tangential stress τ needed to induce macroscopic relative sliding of asperities is determined by the force of friction with the coefficient of friction μ: $\tau=\mu p$, where p is the pressure acting in the considered micro contact. The elastic energy stored in the contact immediately before gross sliding can be estimated as

$$U_{el} \approx \frac{\mu^2 p^2}{2G} D^3 \tag{4}$$

and the criterion (3) is modified as follows:

$$D > \frac{2G \cdot \Delta w}{\mu^2 p^2} . \tag{5}$$

The main difference of this criterion from the classical Rabinowicz' criterion is that the critical asperity size depends not only on material parameters but also on the pressure in that considered asperity. However, even in this case there exists some characteristic asperity size which can be estimated by substituting in (5) the average pressure in micro-contacts, which in good approximation is given by [14]

$$p \approx \frac{1}{2} E^* \nabla z . \tag{6}$$

The criterion for formation of particles thus takes the form

$$D > \frac{\Delta w}{G \mu^2 \nabla z^2} , \tag{7}$$

with the additional constraint of (3), since the pressure cannot exceed σ_0. This criterion does not necessarily assume plastic behaviour and can also be applied to purely elastic media.

2.3. Modified Rabinowicz' criterion for contacts with friction and adhesion

In the paper [15], a contact of two bodies has been considered, which interact by adhesion and friction forces at the same time. In the limit of very strong but short ranged adhesive interactions, it has been shown that the critical force at complete sliding is given by the simple equation

$$F_{x,slip}(a) \approx \mu(F_{N,JKR}(a) + \pi a^2 \sigma_c) \approx \mu \pi a^2 \sigma_c , \tag{8}$$

where $F_{N,JKR}(a)$ is the normal force according to the JKR-theory for the corresponding profile [16, 17]. This can be done if $\pi a^2 \sigma_c / F_{N,JKR}(a) \approx \sqrt{\sigma_c a / (Eh)} >> 1$ which is the case for typical material parameters of metals and junctions larger than about 1 μm. In this case, in the first approximation we can assume that the surfaces are pressed against each other with a constant and high adhesive pressure σ_c. We thus have a contact with the "flow stress" $\tau=\mu\sigma_c$, and Eq. (3) is directly applicable with the only substitution of $\sigma_0=\mu\sigma_c$.

3. WEAR IN SYSTEMS WITH A SOFT SURFACE LAYER

Rabinowicz has formulated his criterion for homogeneous media. However, many tribological systems have a pronounced layered structure – either artificially designed or developed during tribological loading. In the present paper we repeat the arguments of Rabinowicz for such layered systems and find the conditions for plastic smoothing and particle detachment in this case. We follow the presentation of preprint [18].

Consider an elastic medium with elastic shear modulus G_0 covered with a soft elastoplastic layer of thickness h having shear modulus G_c and the tangential yield stress σ_c. This layer can be deposited to the surface artificially or it can appear naturally through mechanically induced chemical reactions of the base material with surrounding substances (lubricant, counter-body, air and so on) [9, 10]. Assume that due to normal loading and tangential sliding a junction with the diameter D is formed, and that the Diameter D is much larger than the thickness of the layer (the opposite case corresponds to a homogeneous medium and is covered by Eq.(3)). The components of the stress tensor in the near surroundings of the junction will be of the order of magnitude of σ_c. If the elastic energy stored in the system is not enough for creating new surfaces with an area of the order of D^2 than the only possible process will be plastic smoothing as illustrated in detail in the paper [13]. In the opposite case, the elastic energy can be relaxed by detaching of a wear particle. In the case of debris formation, two limiting cases are possible:

3.1. Detachment in the base material

In this case, we basically can repeat the line of argument of Rabinowicz. The stored elastic energy has the order of $(\sigma_c^2/2G_0)\cdot D^3$ and the surface energy needed for formation of a wear particle is of the order of $\Delta w\cdot D^2$ where Δw is the work of separation of the base material. The formation of wear particle is possible if $(\sigma_c^2/2G_0)\cdot D^3 > \Delta w\cdot D^2$ or

$$D > \frac{2G_0\Delta w}{\sigma_c^2} \tag{9}$$

which coincides with the Rabinowicz' criterion (3). The only difference from the classical criterion is that the elastic modulus and energy of separation are those of the base material while the critical flow stress is that of the surface layer.

3.2. Detachment inside the surface layer

In this case, the elastic energy which is released due to particle detachment is on the order of $(\sigma_c^2/2G_c)\cdot D^2h$ and the energy needed for detachment $\gamma_c D^2$, where γ_c is the work of adhesion inside the soft layer. The formation of particles is thus possible if $(\sigma_c^2/2G_c)\cdot D^2h > \gamma_c D^2$ or

$$h > h_c = \frac{2G_c\gamma_c}{\sigma_c^2}\,. \tag{10}$$

In this case, the fulfilment of criterion does not depend on the diameter of junction but depends solely on the *thickness* of the layer. *If the thickness of the layer is smaller than the critical one, h_c, then formation of particles is not possible, independently of the size of junctions*. Note that in this case only the properties of the softer surface layer do play a role.

3.3. Criteria for formation of "flat" and "spherical" wear particles

Let us consider in detail the transition between the cases 1 and 2 discussed in the previous Section. Again consider a junction with some particular diameter D. The following cases are possible:

1. $D > \dfrac{2G_0\gamma_0}{\sigma_c^2}$ but $h < \dfrac{2G_c\gamma_c}{\sigma_c^2}$.

In this case, formation of the in-layer particles is not possible but formation of the base-material particles is possible. This is the classical "Rabinowicz case".

2. $D < \dfrac{2G_0\gamma_0}{\sigma_c^2}$ but $h > \dfrac{2G_c\gamma_c}{\sigma_c^2}$.

(This case is possible if the elastic modulus of the surface layer is sufficiently smaller than that of the base material). In this case, the formation of "bulk" particles is not possible but surface-layer flat wear particles can be formed.

3. In the general case, one could suggest the following generalized estimation. Assume that we have a junction of diameter D and detached is a particle of diameter D and thickness h. Then, the elastic energy stored in the system is on the order of

$$U_{el} = \begin{cases} \dfrac{D^2\sigma_c^2}{2}\left(\dfrac{h}{G_c} + \dfrac{H-h}{G_0}\right), & \text{for } H > h \\ \dfrac{D^2\sigma_c^2}{2}\dfrac{H}{G_c}, & \text{for } H < h \end{cases} \tag{11}$$

The energy needed for formation of the above particle is on the order of

$$U_{surf} = \begin{cases} \gamma_0 D^2, & \text{for } H > h \\ \gamma_c D^2, & \text{for } H < h \end{cases} \tag{12}$$

The formation of particles is possible if

$$\begin{cases} \dfrac{D^2\sigma_c^2}{2}\left(\dfrac{h}{G_c} + \dfrac{H-h}{G_0}\right) > \gamma_0 D^2, & \text{for } H > h \\ \dfrac{D^2\sigma_c^2}{2}\dfrac{H}{G_c} > \gamma_c D^2, & \text{for } H < h \end{cases} \tag{13}$$

or

$$\begin{cases} H > \dfrac{2G_0\gamma_0}{\sigma_c^2} - h\left(\dfrac{G_0}{G_c} - 1\right), & \text{for } H > h \\ H > \dfrac{2G_c\gamma_c}{\sigma_c^2}, & \text{for } H < h \end{cases} \tag{14}$$

Let us display these relations graphically on the plane (H, h),

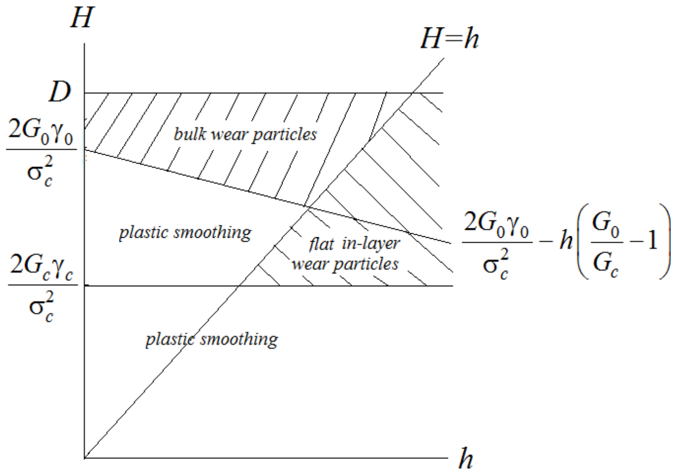

Fig. 2 Schematic representation of conditions given by Eq. (14).

A completely "wear-less" sliding will occur if the following two conditions are fulfilled:

$$h < \frac{2G_c\gamma_c}{\sigma_c^2} \tag{15}$$

and

$$D < \frac{2}{\sigma_c^2}[G_0\gamma_0 - G_0\gamma_c + G_c\gamma_c]\,. \tag{16}$$

Due to softness of the surface layer, the critical junction size can be made large enough so that the only condition which has to be observed would be that given by Eq. (15). This explains the principle of "positive hardness gradient" as condition for wear-less sliding, which was formulated by Kragelsky [3].

Of course, even the process of plastic smoothing will lead to effective "wear" due to "squeezing out" of the surface layer. However, it was shown in [9] (see also [11], §17.5) that in this case the effective wear rate is proportional to the square of the ratio of the layer thickness to the linear size L of the frictional contact zone. The wear coefficient will thus be on the order of

$$k_{adh} \propto \left(\frac{h}{L}\right)^2 \tag{17}$$

and can assume extremely small values. Note that the smaller is the thickness of the surface layer the smaller is the wear coefficient.

4. Asperity Free Concepts for Adhesive Wear

Rabinowicz' criterion is based on the notion of "asperity". An important parameter for application of this criterion is the knowledge of the "asperity size". However, it is widely recognized that the notion of asperity is a poorly defined notion for real surfaces which often have roughness on many length scales. However, without properly defining the size of an asperity, the Rabinowicz' criterion cannot be applied. Looking at a contact configuration of bodies with fractal rough surfaces (Fig. 3), we see more or less continuous clusters of contact areas instead of separated asperities. We would like to suggest a "modified Rabinowicz' criterion" which is largely independent of the definition of an asperity and is based on the ideas first proposed in [19].

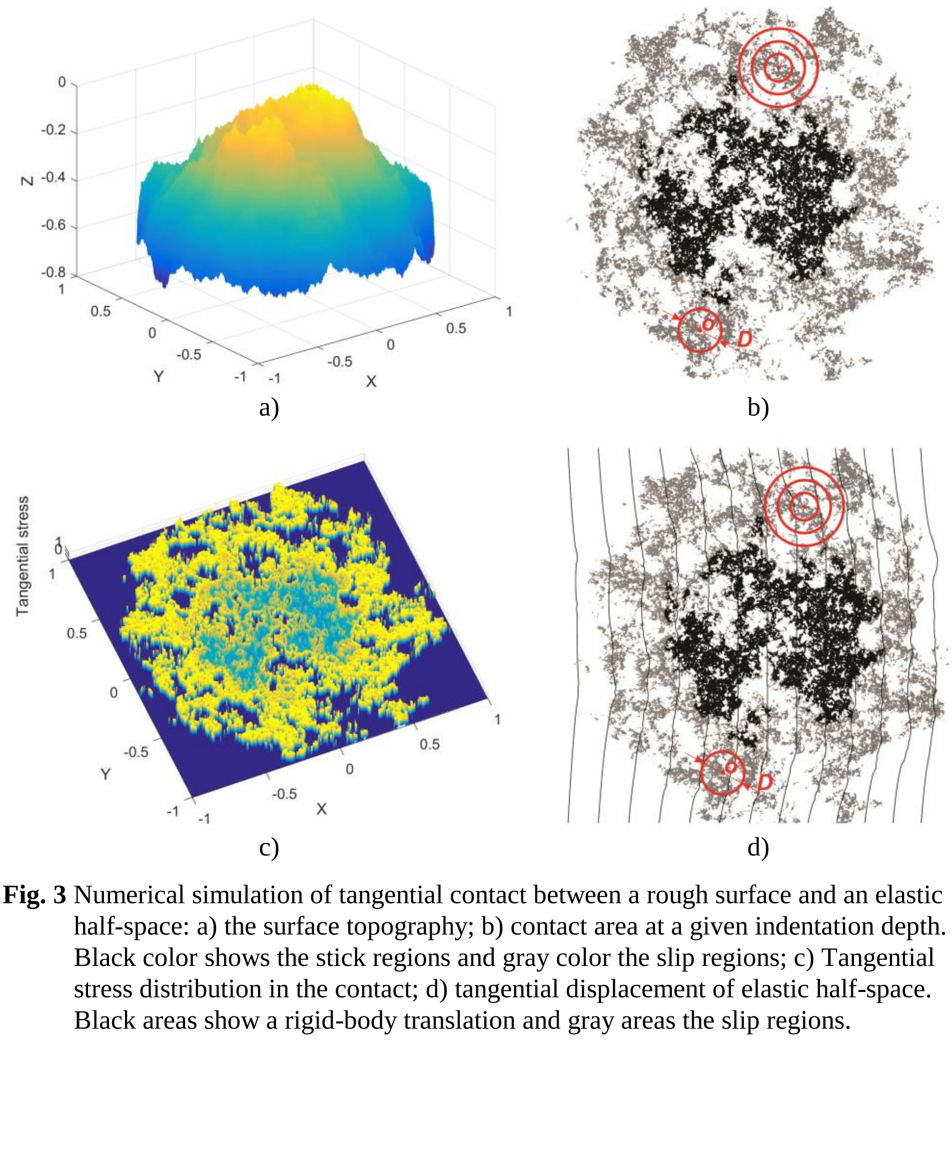

Fig. 3 Numerical simulation of tangential contact between a rough surface and an elastic half-space: a) the surface topography; b) contact area at a given indentation depth. Black color shows the stick regions and gray color the slip regions; c) Tangential stress distribution in the contact; d) tangential displacement of elastic half-space. Black areas show a rigid-body translation and gray areas the slip regions.

Consider as illustration the contact of rough surfaces shown in Fig. 3. A rough sphere with roughness having the Hurst exponent 0.7 was generated according to the rules described in [20]. The indenter was pressed into the elastic half-space and then moved tangentially by a $u_x^{(0)}$ smaller than the displacement corresponding to complete sliding [21]. Both the normal indentation and tangential loading were simulated using the Boundary Element Method as described in [7]. Unlike in [7], we assumed that any two points of bodies are in the stick state as long as the local stress is smaller than a fixed critical value, τ_c. After beginning of slip, the stress remained constant and equal to τ_c, thus mimicking elastic-ideally-plastic behavior in the contact interface. The tangential contact comes into plastic state by overcoming the critical value τ_c which in this context plays the role of the "yield stress" used by Rabinowicz in Eq. (1).

Now consider a circular region centered at an *arbitrary* point with an *arbitrary* diameter D. In Fig. 3b and 3d, several examples of such regions with different positions and different diameters are shown with red circles. The *macroscopic* tangential stress in the selected circular area is equal to $\xi\tau_c$, where ξ is the "filling factor" defined as the ratio of the real contact area in this circle to the area of the circle $\sim D^2$. Assume further, that configuration of the contact in this area corresponds to the plateau of the stiffness, as described in [22]. Then the contact acts as a complete contact. The elastic energy which would be released if a wear particle with the characteristic volume $\sim D^3$ would detach, has then the order of magnitude $((\xi\tau_c)^2/2G)\cdot D^3$. If it is not enough for creating the free surface of the order of D^2, the detaching cannot happen. Thus the criterion for the possibility of detaching a wear particle with the size D is $((\xi\tau_c)^2/2G)\cdot D^3 > D^2\Delta w$ or

$$D > \frac{2G\Delta w}{(\xi\tau_c)^2} . \tag{18}$$

In the most general case, elastic energy that will be relaxed by detaching the considered particle, can be estimated as

$$U_{el} \approx \frac{(\xi\tau_c)^2 D^4}{4Ga_H(D)} \tag{19}$$

where a_H is the Holm-radius of the considered contact configuration [23]. The condition for particle detachment can be written as

$$\frac{D^2}{2a_H(D)} > \frac{2G\Delta w}{(\xi\tau_c)^2} . \tag{20}$$

Generally, the dependence of the Holm-radius on the diameter of the circle can only be determined numerically. Thus, this equation has to be evaluated using numerical simulations of contact and local stiffness of various areas.

Note that for using Eqs. (19) and (20) there is no need to define what an asperity is. By "probing" various positions and diameters, one can identify the material regions which "can potentially produce wear particles". However, in this concept the real contact area will play an essential role so that further ideas may be needed for determination of a robust criterion which does not depend on fine details of the power density of the surface roughness. A very interesting discussion of these aspects can be found in [24].

5. CONCLUSIONS

In the present paper, we applied the Rabinowicz-Molinari criterion for formation of wear particles for a variety of systems differing by the interactions in the interface and by the material properties (elastic and elastoplastic) and structure (homogeneous and layered systems). Of special interest is the result that in the system with a soft layer, no critical size of contact does exist. Instead, there appears some critical thickness. We further discuss a generalization of the Rabinowicz-Molinari criterion for systems with arbitrary complex contact configuration. This formulation does not use the notion of "asperity" and automatically includes "multi-contact" situations.

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